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SIMILARITY OF THE ENERGY-SEPARATION PROCESS IN VORTEX RANQUE TUBES

Sh. A. Piralishvili and A. A. Fuzeeva

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Criteria for estimating the effect of energy separation in a vortex tube have been developed on the basis of the similarity theory and the theory of dimension. The most reliable available experimental data on the process considered have been generalized in the form of criteria equation relating the main characteristics of the energy-separation process in tubes with determined parameters.

Introduction. The effect of energy separation (temperature stratification) and other effects arising in vortex tubes allow them to be widely used for different purposes. However, it is difficult to design a vortex tube with definite integral characteristics for a concrete application because the available experimental data are not clearly understood and there are no entirely correct generalizations. The phenomenon of thermogasdynamic temperature stratification discovered by French engineer-metallurgist J. Ranque almost 70 years ago, which is better known as the effect of energy separation in Ranque–Hilsch tubes or the vortex effect, has found wide-spread use in different fields of engineering and technology, e.g., for designing small-flow-rate vortex tubes for individual air conditioners [1], coolers for automatic-control systems, vortex ejectors, gas-mixture separators, and so on.

The use of vortex tubes in cooling apparatus is limited, as compared to isoentropic turboexpanders. This is explained by the fact that vortex tubes have a low thermodynamic efficiency (27–40%) because a "worthless" differential pressure arises in them in the process of work and they have a low refrigerating capacity.

The advantages of vortex apparatus include the absence of rubbing parts, the large guarantee life, the simple design, the high speed of response, the small dimensions, and the simple principle of operation.

The results of known analytical investigations of the above-indicated phenomenon [1, 2] are contradictory in many respects because of the complexity of the system of equations for a three-dimensional compressible viscous flow. There is no agreement among researchers regarding the possibility of analyzing the Ranque effect with the use of physical models, making its mathematical representation simpler. However, detailed analysis of the existing models of this phenomenon [1, 2] has shown that the hypothesis of vortex–vortex interaction is the most adequate of them.

Previously, most attention has been concentrated on the experimental investigation of the Ranque effect [3–8].

Works devoted to numerical investigation of the Ranque effect are small in number and, judging by the known publications, only approaches to serious investigations are being developed [9].

Experiments on the Ranque effect were conducted, as a rule, in tubes of different configurations, which, at times, prevented the determination of their geometric similarity. Moreover, experimental data were not necessarily generalized from the standpoint of similarity theory and the theory of dimension. The operating parameters of a tube are usually controlled by changing the section of a heated-gas flow, for which purpose a throttle valve is opened or closed. However, there are practically no serious experimental works where the flow section was considered as a controlling factor in the dimension form or as any form parameter. The same can be said about the thermodynamic similarity, elements of which were considered in a number of works [4–8]; however a unique justified solution of the problem has not been found.

Similarity of the Energy-Separation Process. The characteristics of a vortex energy separator are substantially dependent on its geometry, in particular on the shape and open area of the nozzle input, the diaphragm, and the throttle, as well as on the length and diameter of the energy-separation chamber and the angle of taper of the setting.

P. A. Solov'ev Rybinsk Aircraft Technological Academy, 53 Pushkin Str., Rybinsk, Yaroslavskaya Obl., 152934, Russia; email: ana_fuz@mail.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 79, No. 1, pp. 29–34, January–February, 2006. Original article submitted February 17, 2004; revision submitted November 2, 2004.

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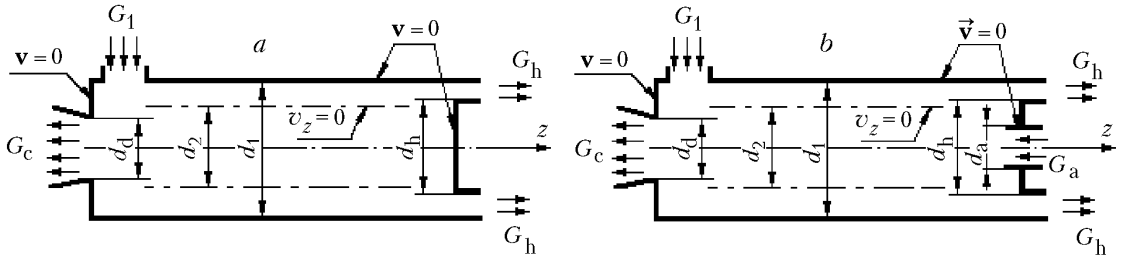


Fig. 1. Types of vortex tubes: a) counterflow tube, $\mu < 1$; b) two-loop tube, $\mu > 1$.

To onvestigate the geometric similarity, we will classify the known vortex tubes, depending on their design, as cylindrical counterflow tubes, diffusion counterflow tubes, and vortex tubes with an additional flow (Fig. 1).

The physical (kinematic or dynamic) similarity of tubes, will be determined using the known system of Navier–Stokes equations for the dynamics of a compressible-viscous-gas motion, as well as the continuity and energy equations. To close the system of equations, it is necessary to determine the effective viscosity and heat conduction of a substance considered. The formulation of the problem is finished by the setting of the boundary and initial conditions as the conditions of adiabaticity and adhesion.

The thermodynamic similarity between substances is determined using reduced equations of their states, which always include several constants characterizing the nature of a given substance. The state equation of a substance can be transformed into a universal dimensionless function of its parameters:

$$\pi_{cr} = \frac{p}{p_{cr}}, \quad \tau_{cr} = \frac{T}{T_{cr}}, \quad \omega_{cr} = \frac{V}{V_{cr}} = \frac{\rho_{cr}}{\rho}, \quad \pi_{cr} = f(\tau_{cr}, \omega_{cr}),$$

e.g., to a reduced Van der Waals equation [10].

It is known that the number of individual constants involved in the state equation of a substance is equal to the number of constants involved in the equation for the potential energy $\varphi(r)$ of interaction between two molecules of this substance. The potential energy of interaction between molecules can be determined as a function of the distance between them in accordance with the Lennard-Jones dependence [11]

$$\varphi(r) = 4\varepsilon_0 \left(\left(\frac{\sigma}{r} \right)^{12} - \left(\frac{\sigma}{r} \right)^{6} \right).$$

The number of the indicated constants exceeds two; consequently, there is no unique state equation for all substances, i.e., there is no law of corresponding states for them [10]. If we find substances for which one or several individual constants are identical or related by a numerical relation common for these substances, the indicated constants will become group constants and, for the considered group of substances, the law of corresponding states will hold. In this case, the substances forming a group will be thermodynamically common.

Researchers have proposed several criteria for determining the dependence of the cooling of a working substance on its physical properties [5–8]. The results of experiments indicate that the adiabatic compression, heat capacity, compressibility, and kinematic viscosity of a working substance depend on the value of the work done by it. However, experiments have shown that not one of the approaches proposed was used for all gases tested. To overcome the contradictions revealed in experiments, it is necessary to develop a complex approach accounting for the maximum possible number of distinctive characteristics of gases.

It has been shown in [5] that the cooling of a substance depends qualitatively on its critical temperature, Boyle temperature, and coefficient *a* in the Van der Waals equation. The physical sense of the coefficient *a* is that it characterizes the value of the molecular attraction. The parameter ε_0 reflects the desired dependence most completely and represents the minimum potential energy of the intermolecular interaction [11]. The electron shells of atoms oscillate continuously relative to the nucleus. These oscillations lead to the appearance of electric fields and Van der Waals

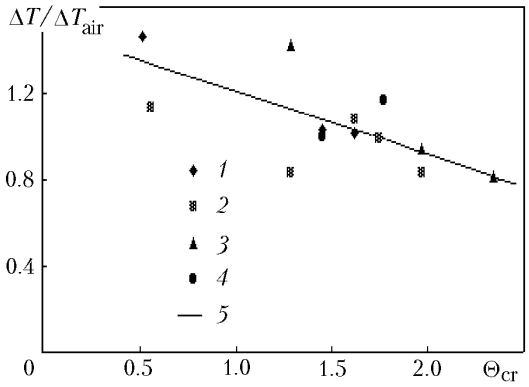


Fig. 2. Dependence of the relative cooling on the reduced temperature: 1) μ = 0.25, π = 6 [6]; 2) μ = 0.5, π = 7 [7]; 3) μ = 0.3, π = 5 [8]; 4) μ = 0.3, π = 5 [5]; 5) theoretical curve $\Theta_{cr} = f(\Delta T/\Delta T_{air})$.

forces, the quantitative measure of which is the coefficient a. The attraction energy is small for gases having a low critical temperature.

The generalization of thermodynamic-similarity characteristics made in [5] does not seem to be entirely correct. First, the argument has the dimensions of temperature and, consequently, cannot be used as a criterion. Second, since the Lennard-Jones potential ε_0, dependent on the properties of a concrete gas, is divisible by a relatively small k that is identical for all gases, the letter can be considered only as a scaling factor. To all appearances, if this regularity is revealed in the process of investigating the thermodynamic similarity, there should exist a constant with the dimensions of temperature that would be unique for each substance. This constant can be the critical temperature T_{cr}.

We introduce the dimensionless parameter

$$\Theta_{cr} = \frac{kT_{cr}}{\varepsilon_0},$$

characterizing the thermodynamic similarity in vortex tubes, which, by analogy with [11], will be called the reduced temperature. In this case, the values of the relative cooling of different gases lie near one straight line (Fig. 2), described by the equation

$$\frac{\Delta T}{\Delta T_{air}} = 1.51 - 0.295\left(\frac{kT_{cr}}{\varepsilon_0}\right).$$

The small scatter of points is explained by the fact that the experimental data presented were obtained for different vortex tubes, the geometric similarity of which cannot be estimated. Since the available experimental data were obtained for a limited number of gases, the challenge was not to determine a correlation between the experimental data obtained but to predict the value of the vortex effect for gases, for which experimental data are absent.

In the present work, we attempted to develop criteria characterizing the effect of energy separation in a vortex tube and to derive, on the basis of generalization of experimental data, a criteria equation relating the main characteristics of the energy separation in tubes with the parameters of these tubes.

Generalization of Experimental Data. Using similarity theory and the theory of dimensions, we obtained a fairly complete set of dimensionless complexes defining the energy separation in a Ranque vortex tube. These complexes were then divided into two groups comprising determining and determined quantities. The determining quantities are initially known parameters: the geometric-similarity parameters f_n, f_d, f_h, and l, the dynamic-similarity parameters π and Re, the kinematic-similarity parameter M, and the dynamic-similarity parameters k, Θ_{cr}, π_{cr}, z_{cr}, τ_{cr}, and Pr. Thus, any determined parameter (μ, μ_h, π_c, π_h, $\bar{\rho}_c$, $\bar{\rho}_h$, $\bar{v}_h$, $\bar{v}_\varphi$, $\bar{v}_z$, Θ_c, Θ_h) can be represented as a function of determining parameters. For example, the relative cooling temperature is uniquely dependent on the following parameters:

$$\Theta_c = f(\bar{f}_n, \bar{f}_d, \bar{f}_h, \bar{l}, \pi, M, k, \Theta_{cr}, \pi_{cr}, \tau_{cr}, z_{cr}, Re, Pr) .$$

When the data obtained are compared with the data of [4, 5], it is apparent that new quantities accounting for the thermodynamic properties of gases and the relative open area of the throttle appeared. Thus, in order for the energy-separation effects for one gas to be predicted on the basis of the data for another gas, it is necessary that these gases be thermodynamically similar and, what is more, be in one and the same group of substances.

For generalization of experimental data and representation of them in the form of criteria dependences, such as

$$y = a_0 y_1^{a_1} y_2^{a_2} \ldots y_n^{a_n}$$

we developed a program (written in the Turbo Pascal language) for data processing with the use of the least-squares technique.

Since the dependence $\Theta_c = f(\mu)$ has, when all the other parameters are constant, a clearly defined extremum, it will be represented as the product

$$\Theta_c = a_0 \mu^{a_1} (1 - \mu)^{a_2} ,$$

which should decrease the error of the formulas obtained.

The thermodynamic similarity was calculated with the use of the most complete data [6] on the thermodynamic parameters. However, the experimental data were processed and the criteria equation was derived based on certain assumptions because not all determining parameters were defined. For example, the gas outflow from the nozzle input was assumed to be critical (M = 1), which is permissible at $\mu \le 0.6$–0.7, and was not taken into account in the calculations, and the Reynolds number was calculated from the following dependences because the flow rate was not experimentally determined:

$$Re = \frac{4G_1}{\pi d_1 \eta_{T_1}} = \bar{f}_n \gamma \left(\frac{2}{\gamma+1}\right)^{\frac{3-\gamma}{2(\gamma-1)}} \frac{d_1 p_1}{\eta_{T_1}\sqrt{\gamma \frac{p_1}{\rho_1}}} = \bar{f}_n \gamma \left(\frac{2}{\gamma+1}\right)^{\frac{3-\gamma}{2(\gamma-1)}} \frac{d_1 c_1}{\eta_{T_1}} = \bar{f}_n \gamma \left(\frac{2}{\gamma+1}\right)^{\frac{3-\gamma}{2(\gamma-1)}} Re_{c_1} .$$

After the processing of the data on the operation of a tube with gases of different physical nature [7], we obtained the dependence

$$\Theta_c = 0.967 (1 - \mu)^{-0.146} \mu^{-0.048} \pi_{cr}^{-0.2} \tau_{cr}^{0.174} z_{cr}^{0.278} k^{-0.211} Re^{-0.098} \Theta_{cr}^{0.023} Pr^{-0.0003} .$$

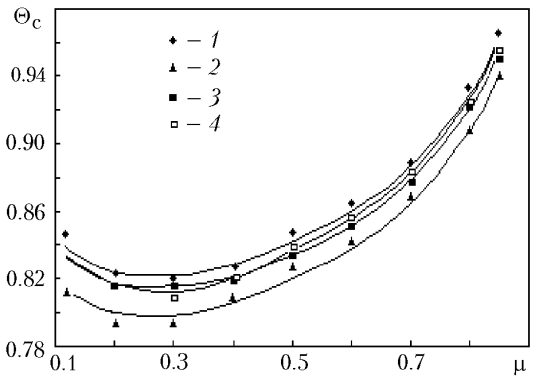
The relative error of this dependence is not larger than 5.21%. Detailed analysis has shown that the largest contribution to the error is made by helium. When it is excluded from the data processed, the dependence takes the form

$$\Theta_c = 0.783 (1 - \mu)^{-0.137} \mu^{-0.051} \pi_{cr}^{0.097} \tau_{cr}^{-0.089} z_{cr}^{0.035} k^{0.034} Pr^{0.0036} \Theta_{cr}^{0.017} Re^{0.0018}$$

with a relative error not larger than 1.7%, a maximum deviation of 0.0145, and an average deviation of 0.0067. Then we considered only the thermodynamic-similarity parameters. In this case, the above-indicated dependence takes the form

$$\Theta_c = 1.044 (1 - \mu)^{-0.137} \mu^{-0.051} \pi_{cr}^{-0.037} \tau_{cr}^{0.015} z_{cr}^{0.134} k^{-0.325} Pr^{-0.02} \Theta_{cr}^{-0.037}$$

with a relative error of 1.7%. Thus, the exclusion of the Reynolds number from consideration does not increase the error of the computational formula. This relation is true for argon, hydrogen, oxygen, and air at $\mu = 0.12$–0.85, an initial pressure $p_1 = 4.5$, and a temperature $T_1 = 330$ K.



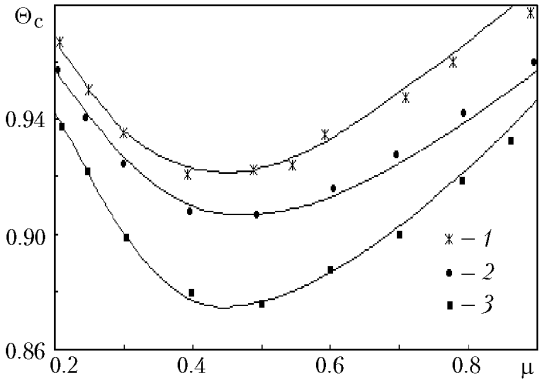


Fig. 3. Comparison of the data calculated by the regression dependence with the experimental data [7] obtained for different gases: 1) air; 2) hydrogen; 3) argon; 4) oxygen.

Fig. 4. Comparison of the data calculated by the criteria dependence with the experimental data [12] obtained for tubes of different diameter: d_1 = 5 (1), 10 (2), and 15 mm (3).

Calculation and experimental dependences of Θ_c on μ are presented in Fig. 3. Experiments were conducted in a tube with the following geometric parameters: f_n = 0.1, f_d = 0.25 (d_d = 0.5), l = 9, α = 0, and the size of the spider at the hot end D = 8. The differential pressure was equal to π = 6 and the initial pressure and temperature were p_1 = 4.5 and T_1 = 330 K. Since all the parameters were held constant, it was difficult to compare the characteristics studied at π_{cr} = idem and τ_{cr} = idem. Since the experimental data obtained give no information on the influence of the geometry and the differential pressure π on the characteristics of tubes, it is necessary to perform additional experiments to introduce these parameters into the regression dependence.

In [12], the results of experiments in small-size conic vortex tubes of diameter 5, 10, 15, and 20 mm with a cone angle of the energy-separation chamber $\alpha = 3^o$, a relative diameter of the diaphragm aperture d_d = 0.58, a relative length of the energy-separation chamber l = 14, and a relative area of the nozzle input f_n = 0.183, 0.112, 0.152, and 0.111 are presented. Having generalized the experimental data, we obtained the following dependence:

$$\Theta_c = 0.619\,(1-\mu)^{-0.083}\,\mu^{-0.084}\,\bar{f}_n^{-0.09}\,d_1^{-0.03}\,\pi^{-0.04}\ ,$$

where the diameter of the tube was measured in millimeters.

In Fig. 4, the data calculated by the criteria dependence are compared with experimental data. The relative error given by this dependence is no more than 2.45%, the maximum deviation is 0.023, and the average deviation is 0.0074. The indicated dependence was obtained for μ = 0.18–0.82, π = 2–7, d_1 = 5–20 mm, and f_n = 0.111–0.183.

It is easy to see that the dependence obtained allows one to determine the relative cooling with a high accuracy, even though it does not involve all similarity criteria. Safonov et al. [12] did not describe the experimental process completely; therefore, the criteria of kinematic M and dynamic Re similarity and the open area of the throttle f_h were not taken into account.

Conclusions. An approximate dependence for calculating the cooling of a gas by the experimentally determined cooling of air has been derived on the basis of analysis of the thermodynamic properties of actual gases and data on the blow-through of vortex tubes. To do such calculations, it is necessary to know the critical temperature and the Lennard-Jones potential of a gas. Experimental data fit with a sufficient accuracy in the theoretical curve obtained.

A regression equation for calculating the relative cooling of a gas by its thermodynamic parameters has been derived. However, this equation was obtained with no account for the geometry of a vortex tube and the differential pressure π in it because these quantities were held constant in the experiment. To derive a more general regression equation with account for all determining parameters, it is necessary to perform additional experimental investigations.

A regression dependence for calculating the relative cooling of a gas has also been obtained on the basis of analysis of the experimental data obtained for a number of small-size conic vortex tubes of diameter 5, 10, 15, and 20

mm. This dependence accounts for some geometric parameters; however, it does not take into account the criteria of kinematic M and dynamic Re similarity and the open area of the throttle $\bar{f}_h$.

There are a large number of experimental data on the vortex effect in the literature; however, more than 90% of these data cannot be completely processed because of the shortage of measured parameters. For example, in the overwhelming majority of cases, the data on the flow rate of a gas, the geometry of a spider, and others are absent.

NOTATION

a, Van der Waals coefficient; c, velocity of propagation of perturbations, m/sec; D, characteristic size of a spider, mm; d, diameter of the characteristic cross section of a flow, m; f, area of the flow section, m^2; G, flow rate, kg/sec; k, Boltzmann constant, J/K; l, length of the energy-separation chamber, m; M, Mach number; p, pressure, Pa; Pr, Prandtl criterion; r, intermolecular distance, m; Re, Reynolds criterion; T, temperature, K; V, volume, m^3; v, velocity of a gas flow, m/sec; $\mathbf{v}$, velocity vector, m/sec; y, variable; z, Young criterion; α, cone angle of the energy-separation chamber, deg; ε_0, Lennard-Jones potential, J; γ, adiabatic index; η, dynamic viscosity, Pa; φ, potential energy of intermolecular interaction, J; μ, relative fraction of a flow; π, decrease in pressure; Θ, relative temperature; ρ, density, kg/m^3; σ, intermolecular distance at which $\varphi(r) = 0$, m; τ, reduced temperature; ω, reduced volume. Subscripts: 1, input parameters; 2, parameters at the radius separating vortices; overscribed bar, relative value; r, φ, z, axes of the cylindrical coordinate system; air, air; h, heated flow; d, diaphragm; a, additional flow; cr, critical parameters; n, nozzle; c, cooled flow.

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